Technical Note

Designing for Inertia

Inertia is resistance to a change in motion. It is determined by an object's size, shape, and mass. The greater the inertia, the greater is the amount of force (torque) required to change the object's state of motion.

During acceleration, the motor must produce enough torque to accelerate both its rotor and the load. An inertia acceleration factor, J_{ratio}, is the ratio between the load inertia and the motor inertia. It is an important factor that determines system stability and reducer service factor. If J_{ratio} is too high, the motor cannot stabilize the load and the application has 'inertia mismatch'.

When sizing a gearmotor for an application, it is important knowing how to calculate J_{ratio}, how to use J_{ratio}, and how to reduce J_{ratio}.

Calculating J_{ratio}

By definition,

$$J_{ratio} = \frac{\text{Total System Inertias}}{\text{Total Motor Inertias}} = \frac{J_{red} + J_X}{J_m + J_z},$$

Where,

J_{red} = Reducer inertia (available from SEW)

J_x = Reflected load inertia (measured at the motor shaft). The following equations define J_x, depending upon the type of motion.

Linear Motion (Conveyor, Travel Drive, Hoist)	Rotational Motion (Turntable, Solid Cylinder)
$J_x = \frac{W_{total}}{39.5} \times \left(\frac{v_{max}}{n_m}\right)^2$	$J_x = \frac{W_{total}}{2} \times \left(\frac{D_t}{2 \times 12}\right)^2 \times \left(\frac{n_t}{n_m}\right)^2$

Where,

W_{total} = Total weight (lbs)
v_{max} = Maximum velocity (fpm)
D_t = Turntable diameter (inches)
n_t = Turntable speed (rpm)
n_m = Motor full-load speed (rpm)

J_m = Motor inertia (from SEW Gearmotor Catalog – examples shown below)

Motor	Hp	Motor Inertia (lb-ft²)	
		Without Brake	With Brake
DT71K4	0.25	0.0062	0.0084
DT90L4	2	0.0789	0.0936
DV132S4	7.5	0.346	0.375
DV180L4	30	3.064	3.316

J_z = Inertia of cast iron Z-fan (from SEW Drive Engineering Volume 1):

Motor Frame Size	Z-Fan Inertia (lb-ft²)	Motor Frame Size	Z-Fan Inertia (lb-ft²)
71	0.0475	**112**	0.428
80	0.0713	**132S**	0.513
90	0.2380	**132M/ML 160M**	1.187
100	0.3180		

Example 1:
Calculate the J_{ratio} for the following conveyor (linear motion) application:

Load weight	3200 lbs
Belt weight	800 lbs
Max velocity	180 fpm
Gearmotor Model:	K67DV132S4BMG8 (J_{red} = 0.0102 lb-ft², ratio = 9.66:1)

Solution:
The model number indicates that the motor has a brake (BMG8), but not a Z-fan (no "Z"). Therefore, from the chart above, J_M = 0.375 and J_Z = 0.

$$J_X = \frac{W_{total}}{39.5} \times \left(\frac{v_{max}}{n_m}\right)^2 = \frac{3200\text{ lbs} + 800\text{ lbs}}{39.5} \times \left(\frac{180\text{ fpm}}{1720\text{ rpm}}\right)^2 = 1.109\text{ lb-ft}^2$$

$$J_{ratio} = \frac{J_x + J_{red}}{J_m + J_Z} = \frac{1.109\text{ lb-ft}^2 + 0.0102\text{ lb-ft}^2}{0.375\text{ lb-ft}^2 \; + \; 0} = \frac{1.119}{0.375} = 2.98$$

The J_{ratio} indicates that the load inertia is 2.98 times greater than the motor inertia. Consequently, it requires nearly 3 times more torque to accelerate the load than to accelerate the motor.

Using J_{ratio} – Application without Inverter (Constant Speed)

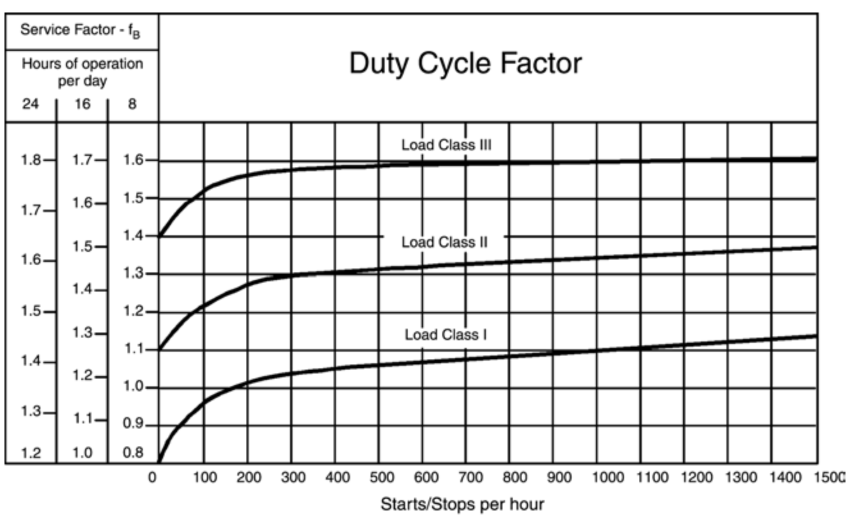
Without an inverter, a motor produces full starting torque across the line – typically 250% to 300% - for as long as it takes the motor to accelerate to its rated speed. The higher the inertia, the longer is the acceleration period. Each time the motor starts and stops, its high starting torque induces shock and fatigue to the reducer. Thus, to ensure the reducer's longevity, SEW recommends a reducer service factor, f_B, based upon the following parameters:

- J_{ratio}
- Load classification
- Number of starts and stops per hour
- Hours of operation per day

The value of J_{ratio} determines a Load Class, per the following table. Notice how the Load Class (shock load) increases as J_{ratio} increases. The increase occurs because a high inertia load consumes most of the starting torque. As a result, more torque goes through the reducer to start the load than to start the motor, producing more "shock" to the reducer.

J_{ratio}	Load Class	Description
$0 < J_{ratio} \leq 0.2$	I	Uniform load
$0.2 < J_{ratio} \leq 3.0$	II	Moderate shock load
$3.0 < J_{ratio} \leq 10$	III	Heavy shock load
$J_{ratio} > 10$	Contact SEW	

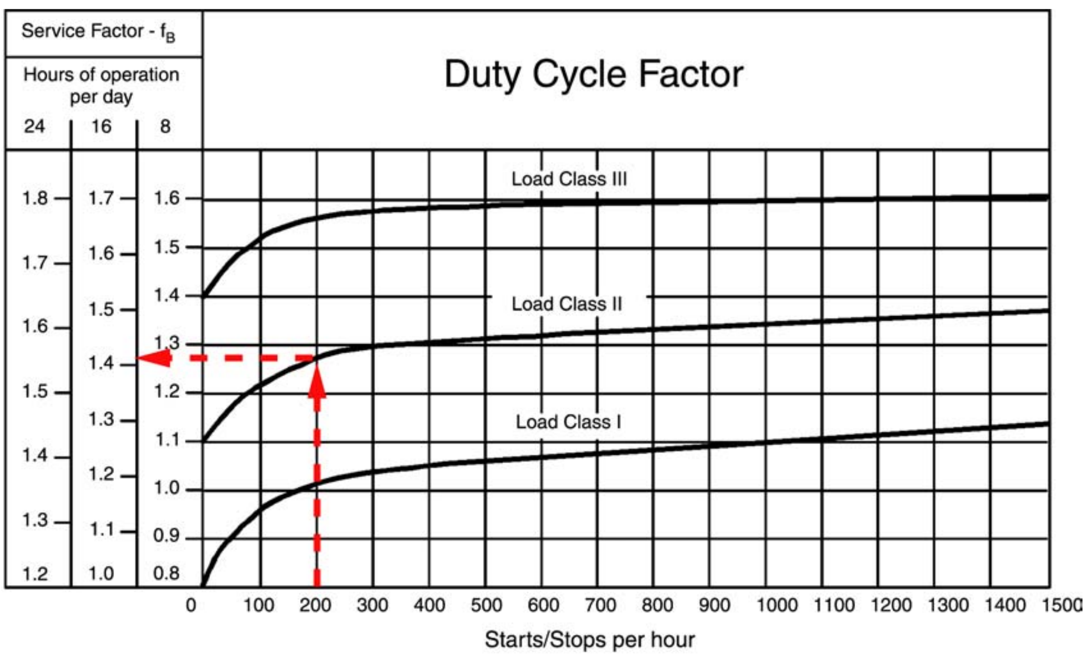
The load class, hours of operation, and the starts/stops per hour determine the service factor, f_B, in the graph below. Note: the load class also determines the recommended tolerance for a customer's solid shaft when used with a hollow shaft reducer. (See Technical Note GM-025 for more information on tolerances.)



Example 2:
Is an R57DV132S4 (i = 9.35) an acceptable selection for the following application?

- 200 starts/stops per hour
- 14 hours/day
- J_{ratio} = 2.87

Solution:
Since the J_{ratio} lies between 0.2 and 3.0, the table on the previous page indicates that the Load Class = II. Thus, using the chart below, the recommended service factor, f_B , is greater than or equal to 1.4



From the SEW Gearmotor Catalog, an R57DV132S4, 9.35:1, contains a service factor equal to 1.3. Since the application requires a service factor greater than 1.4, the reducer is too small and unacceptable.

However, an R67DV132S4, 8.70:1, with a service factor equal to 1.7 is an acceptable selection.

Using J_{ratio} – Application with Inverter (Variable Speed)

The procedure used to size a gearmotor for an application with an inverter differs from the procedure used for an application without an inverter. Without an inverter, the gearbox must withstand the motor's high starting torque. The maximum torque is unknown. Therefore, as previously shown, the J_{ratio} determines a service factor that guides the designer in selecting a reducer with enough torque capacity to withstand the shock imposed by the starting torque.

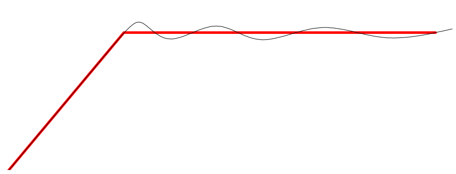
However, in an application that uses an inverter to control various movements, the inverter limits the starting torque (typically 150%). Since there are often multiple movements involving different torques and since the starting torque is lower, the designer must first calculate the maximum torque of the system, then select a reducer with a torque capacity greater than or equal to this maximum torque. An additional service factor is unnecessary unless the designer wishes to use the next size gearbox for its larger bearings and its longer bearing life.

Caution: When an inverter that controls an SEW brakemotor encounters a power loss or an emergency stop, the brake sets immediately. Depending on the brake torque, a sudden stop may act as a shock to the reducer. The following guidelines assist in correcting this situation.

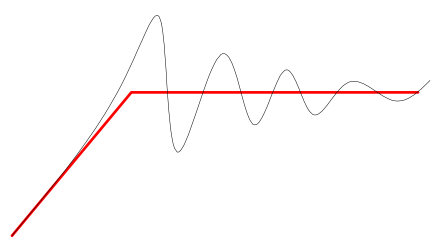
1. Consider the brake torque, not just the motor starting torque, when calculating the maximum torque of the system.
2. Apply an additional service factor to the reducer to accommodate the shock.
3. Reduce the brake torque to less than 150% of the motor's nominal torque by using alternate brake springs.
4. Use a fast, controlled stop instead of an uncontrolled stop on the inverter during an emergency stop condition.

J_{ratio} is important in an inverter application because it determines if the desired acceleration rate is obtainable. When the load inertia is very large compared to the motor inertia (i.e.: large J_{ratio}), the motor has difficulty controlling the load during speed changes. The following problems may arise.

- The motor pinion shaft breaks.
- Position accuracy is poor.
- The motor experiences difficulty when stopping the load.
- After acceleration, the motor experiences difficulty stabilizing the load at a constant speed. Large overshoot occurs, as shown below.



Minimal Overshoot – Good Control



Large Overshoot – Poor Control

The following table shows acceleration guidelines to maximize the system stability.

	J_{ratio}		a (ft/sec²)
If	$20 < J_{ratio} \leq 35$	then	$a \leq 1.64$
	$15 < J_{ratio} \leq 20$		$a \leq 4.92$
	$10 < J_{ratio} \leq 15$		$a \leq 8.2$
	$J_{ratio} \leq 10$		a = not limited

Reducing J_{ratio}

When the J_{ratio} is too high, there are two methods to reduce it.

1. Lower the Reflected Load Inertia

   Load inertias reflected to the motor shaft change by the square of the overall external ratio. Therefore, doubling the gear ratio changes the reflected load inertia by one-fourth (¼). Since the speed of the application is usually unchangeable, the only way to increase the ratio is to increase the input speed, as well, by one of the following methods.

   - Use an inverter and increase the gearbox ratio (or other external ratio) by 33-50%. Also, set the maximum operating frequency to 80 or 90 Hz. A disadvantage to this method is that when the frequency increases while the voltage remains constant (above 60Hz in North America), the breakdown torque decreases – a characteristic that is commonly referred to as "field weakening". However, the reduction is usually acceptable for maximum frequencies below 90 Hz.

   - Use an inverter and double the gearbox ratio (or other external ratio). "Supercharge" the motor by using a 460-volt inverter with a motor connected at low voltage (230V). The result is a motor that receives 230 volts at 60 Hz and 460 volts at 120 Hz. Since the voltage-to-hertz ratio remains constant, the motor does not experience field weakening.

     Note: When supercharging the motor, size the inverter according to the motor's HP rating at 120 Hz, which is double the rating at 60Hz! For example, use a 2 HP, 460V inverter with a 1 HP, 230V motor.

   - Use a motor with fewer poles and increase the gearbox ratio. For example, change from a 4-pole motor (1800 rpm) to a 2-pole motor (3600 rpm) and double the gearbox ratio. Caution: stopping the load with a brakemotor at 3600 rpm may dramatically lower the brake life! Therefore, using an inverter with a braking resistor may be a better solution.

Example 3:
Using Example 1 (page 2) and the following reducers, compare their reflected load inertias.

- K67DV132S4, 180 fpm, 9.66:1, 60 Hz max
- K67DV132S4, 180 fpm, 19.30:1, 118 Hz max (with inverter)

Solution:

9.66 : 1 – 60 Hz	19.30 : 1 – 118 Hz
$J_x = \frac{W_{total}}{39.5} \times \left(\frac{v_{max}}{n_m}\right)^2$ $= \frac{3200\text{ lbs} + 800\text{ lbs}}{39.5} \times \left(\frac{180\text{ fpm}}{1720\text{ rpm}}\right)^2$ $= \frac{4000}{39.5} \times (0.10465)^2$ $= 1.11\text{ lb-ft}^2$	$J_x = \frac{W_{total}}{39.5} \times \left(\frac{v_{max}}{n_m}\right)^2$ $= \frac{3200\text{ lbs} + 800\text{ lbs}}{39.5} \times \left(\frac{180\text{ fpm}}{3460\text{ rpm}}\right)^2$ $= \frac{4000}{39.5} \times (0.052)^2$ $= 0.274\text{ lb-ft}^2$

1.11 / .274 = 4.0.

Therefore, doubling the ratio and doubling the input speed changes the reflected inertia by a factor of **4.0**

2. Increase the Total Motor Inertia

Since the total motor inertia is in the denominator of the J_{ratio} equation, increasing it decreases the J_{ratio} and adds stability to the system. The following options increase the total motor inertia and may be used separately or together.

- Adding a cast iron Z-fan to the motor
- Using a larger motor

Note: Increasing motor inertia will also affect the acceleration rate, which may be unacceptable. Adding a Z-fan decreases the acceleration rate. Using a larger motor may either increase or decrease the acceleration, depending on the motor. A larger motor has more inertia that reduces acceleration. However, it also has more starting torque that increases acceleration. Sometimes the best solution is to add a Z-fan and use a larger motor.

Example 4:
The following indexing application is a travel car. Check the system for stability and increase the motor inertia, if necessary.

Load weight	5000 lbs
Car weight	200 lbs
Max velocity	250 fpm
Acceleration	8.56 ft/s^2 (from calculations)
R57DT100L4BMG4	(J_{red} = 0.0067 lb-ft^2, J_m = 0.139 lb-ft^2, ratio = 9.35:1)

Solution:

$$J_X = \frac{W_{total}}{39.5} \times \left(\frac{v_{max}}{n_m}\right)^2 = \frac{5000\,\text{lbs} + 200\,\text{lbs}}{39.5} \times \left(\frac{250\,\text{fpm}}{1680\,\text{rpm}}\right)^2 = 2.9\text{ lb-ft}^2$$

Therefore,

$$J_{ratio} = \frac{J_x + J_{red}}{J_m + J_Z} = \frac{2.9\text{ lb-ft}^2 + 0.0067\text{ lb-ft}^2}{0.139\text{ lb-ft}^2 + 0} = \frac{2.9067}{0.139} = 20.9$$

According to the table on page 6, when the acceleration is 8.56 ft/s^2, the J_{ratio} should be less than 10 for system stability. Thus, a value of 20.9 indicates that the system is unstable. In order to decrease it, add inertia from either a Z-fan or from the next size motor.

With Z-Fan and same motor (DT100L4BMG4Z):

$$J_{ratio} = \frac{J_x + J_{red}}{J_m + J_Z} = \frac{2.9\text{ lb-ft}^2 + 0.0067\text{ lb-ft}^2}{0.139\text{ lb-ft}^2 + 0.318} = \frac{2.9067}{0.457} = 6.36$$

With larger motor, DV112M4BMG8:

$$J_{ratio} = \frac{J_x + J_{red}}{J_m + J_Z} = \frac{2.9\text{ lb-ft}^2 + 0.0067\text{ lb-ft}^2}{0.262\text{ lb-ft}^2 + 0} = 11.1$$

The J_{ratio} of the larger motor is close to 10, but still unacceptable. However, adding a Z-fan reduces the J_{ratio} considerably and ensures the best stability.

Caution: The reader may think to use a servomotor to increase system stability. However, there are pros and cons to using a servomotor. Since it has less inertia (J_m) than an induction motor, J_{ratio} actually increases. However, if a high-speed servomotor (such as 3000 or 4000 rpm) is used, then the system inertia (J_x) decreases considerably, which decreases J_{ratio}. Therefore, the only way to know the overall effectiveness of a servomotor is to calculate J_{ratio}.